

# Coherent mixing in three and four quark generations

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## Abstract

New dynamical mechanism of quark mass generations and mixing is demonstrated in the examples of three and four generations. In the framework of the new mixing pattern, called the coherent mixing, the CKM elements are predicted compatible with experimental data for three generations, and are strongly constrained for four generations.

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1. The idea that quark masses can be obtained from composite scalar fields instead of elementary Higgs fields is actively developed last decades, for reviews and references see [1, 2].

The basic underlying mechanism for the spontaneous electroweak symmetry breaking (EWSB) and mass generation was the effective quartic fermion interaction which is bosonized in the standard way. What is left unanswered in this approach is the dynamical origin of generations and the peculiar pattern of masses and mixings in the observed up to now three quark generations.

It is a purpose of the present letter to describe, basing on the recent paper [3], the mechanism which produces three or more generations of quarks and can predict intervals of values for mixing coefficients and masses of  $t', b'$  in the case of the fourth generation.

One starts with the unbroken gauge field  $C_\mu$  (it can be taken as  $SU(N)$  field, but this is not important for what follows) and quark Lagrangian  $L = g\bar{\psi}_a\gamma_\mu C_\mu^{ab}\psi_b$ , where fields are organized into two sectors  $\psi_a^{(i)}, i = 1, 2$ .

$$\psi_{Li}^{(1)} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}; \psi_R^{(1)} = (d_R, s_R, b_R) \quad (1)$$

$$\psi_{Li}^{(2)} = \begin{pmatrix} d_L^c \\ u_L^c \end{pmatrix}, \begin{pmatrix} s_L^c \\ c_L^c \end{pmatrix}, \begin{pmatrix} b_L^c \\ t_L^c \end{pmatrix}; \psi_R^{(2)} = (u_R^c, c_R^c, t_R^c)$$

where  $\psi^c$  means the charge conjugated quark field.

We assume that nonperturbative vacuum fields  $C_\mu$  can form connected field correlators, such that (vacuum averaging of Euclidean fields is implied)

$$tr \ll C_{\mu_1}(x_1)C_{\mu_2}(x_2)...C_{\mu_n}(x_n) \gg \equiv J_{\mu_1,...\mu_n}^{(n)}(x_1, x_2, ...x_n). \quad (2)$$

We note, that the form (2) is in general not gauge invariant, but for non-confining field  $C_\mu(x)$  one can define (2) in such a way, that in the local limit  $|x_i - x_j| \rightarrow 0, i, j, = 1, ...n$  one can isolate in (2) a gauge invariant piece of the quark self-energy (this implies that the scale  $M$  of  $J^{(n)}$  can be taken much larger than any other mass in the problem). As a result one obtains in the partition function  $Z = \int DCD\psi D\bar{\psi}e^L$ ; the effective quark Lagrangian

$$\langle \exp \int L d^4x \rangle = \exp \left\{ \sum_{n=2,4,6,...} J_n(x_1, ...x_n) \Psi(x_1) ... \Psi(x_n) dx_1 ... dx_n \right\}. \quad (3)$$

Here  $\Psi(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$ , and we have suppressed spinor and group indices.

Now making pairwise Fierz transformations and keeping only scalar and pseudoscalar terms, one can express all quark bilinears in terms of  $\Phi_{RL}$  and  $\Phi_{LR}$ , where  $\Phi_{RL}(x_1, x_2) = \bar{\psi}_R^a(x_1)\psi_L^a(x_2)$ , superscript  $a$  denotes  $SU(2)$  index of EW group or its GUT equivalent. Next one does a bosonization trick introducing auxiliary functions  $\mu(x_i, x_k), \varphi(x_i, x_k)$  in  $\delta$ -function terms and finding the stationary points of (3) in  $\mu$  and  $\varphi$

$$\begin{aligned} \langle \exp \int L d^4x \rangle_C &= \int D\varphi D\varphi^+ \tilde{\delta}(\varphi - \Phi_{RL}) \tilde{\delta}(\varphi^+ - \Phi_{LR}) \times \\ &\times \exp \left\{ - \sum_{n=1,4} \int J_n(\varphi\varphi^+ + \varphi^+\varphi)^{n/2} dx_1 ... dx_n \right\}, \end{aligned} \quad (4)$$

where  $\tilde{\delta}(f - F) = \int D\mu \exp \left\{ -i \int \mu(f - F) dx \right\}$ .

The final equation can be written in the momentum space for the quark mass function  $\mu(p)$  in the form (see [3] for derivation and more discussion)

$$\mu(p) = \sum_n \int D_n \{p_1, ...p_{n-1}\} \bar{J}_n \{p; p_1, ...p_{n-1}\}. \quad (5)$$

Here  $D_n\{\dots\}$  denotes product  $\prod_{k=1}^{n-1} \frac{d^4 p_k}{(2\pi)^4} d(p_k)$ ,  $d(p) = \frac{\mu(p)}{p^2 + \mu^2(p)}$ . Note, that  $\mu(p)$  plays the role of the  $p$ -dependent mass of quark, since it enters the quark Green's function as  $S^{-1}(p) = \hat{p} + i\mu(p)$ .

It is important, that the stationary points of the wave functional (4) can be written for the real quantity  $\hat{\varphi}(x_i, x_k) \equiv -i\varphi(x_i, x_k)$  in the local limit ( $M \rightarrow \infty$ ) as follows

$$(-\square + \mu^2)\hat{\varphi} = -\frac{\delta U\{\hat{\varphi}\}}{\delta \hat{\varphi}(x)}; \quad U\{\hat{\varphi}\} = -\frac{1}{2}\tilde{J}_2\hat{\varphi}^2 + \frac{1}{4}\tilde{J}_4\hat{\varphi}^4 - \frac{1}{6}\tilde{J}_6\hat{\varphi}^6 + \dots \quad (6)$$

At this point one realizes that  $\tilde{J}_n$  are positive for connected correlators and therefore  $U\{\hat{\varphi}\}$  can have multiple minima  $\hat{\varphi}_i$ . We can associate composite scalars of generations with these minima, and then solutions of (6) of kink type, which connect different minima  $i, k$  of  $U\{\hat{\varphi}\}$  can be related to mixing solutions of composite scalars  $\hat{\varphi}_{ik}$ . The corresponding masses  $\mu_{ii} \equiv \mu_i$  and  $\mu_{ik}$  can be found from the equation  $\hat{\mu} = (\square + \hat{\mu}^2)\hat{\varphi}$  where both  $\hat{\mu}$  and  $\hat{\varphi}$  are matrices in generation indices. In this way one obtains the initial mass matrix  $\mu_{ik}$ , which should be diagonalized to yield the final physical mass matrix  $\hat{m}$  and CKM mixing coefficients  $V_{ik}$ . We shall not make explicit at this point the coefficients  $\tilde{J}_n$  and the functional  $U\{\hat{\varphi}\}$ , referring it to later publications, but rather shall try to guess the form of the matrix  $\hat{\mu}$ , corresponding to the realistic physical masses and CKM coefficients.

Namely, because of the strong hierarchy of masses,  $\mu_{33} \gg \mu_{22} \gg \mu_{11}$ , the matrix solution of (5)  $\mu_{ik}$  acquires the approximate form  $\mu_{ik} = \sqrt{\mu_i \mu_k}$ , with  $\mu_i \equiv \mu_{ii}$  and  $\mu_k \equiv \mu_{kk}$ , and one can write a slightly distorted form

$$\mu_{ik} = \sqrt{\mu_i \mu_k (1 + \eta_{ik})}, \quad i \neq k; |\eta_{ik}| < 1. \quad (7)$$

We shall call (7) as in [3] the Coherent Mixing Mechanism (CMM), and apply it to the case of three and four generations.

2. In the case of three generations one can expand the eigenvalue equation  $\det(\hat{\mu} - m\hat{1}) = 0$  to the lowest orders in  $\eta_{ik}$  to find the physical masses  $m_i$

$$m^3 - m^2\sigma + m\xi - \zeta = 0 \quad (8)$$

with  $\sigma = \sum_{i=1}^3 \mu_i$ ,  $\xi = -\sum_{i \neq j} \mu_i \mu_j \eta_{ij}$ ,  $\zeta = \mu_1 \mu_2 \mu_3 \left( -\frac{1}{4} \sum_{i \neq j} \eta_{ij}^2 + \frac{1}{2} \sum_{i \neq j} \sum_{l \neq k} \eta_{ij} \eta_{lk} \right)$ .

For  $m_1$  to be positive and for the natural hierarchy  $m_1 \ll m_2 \ll m_3$ , one needs  $\eta_{ik} < 0$ ,  $i, k = 1, 2, 3$  and choosing  $\eta_{13} = \eta_{23} \equiv -\eta$ ,  $\eta_{12} = -\delta$ ,  $0 <$

$\delta \ll \eta$ , one has

$$m_1 \approx \mu_1 \delta, \quad m_2 \approx \frac{\mu_3}{m_3}(\mu_1 + \mu_2)\eta, \quad m_3 \approx \mu_1 + \mu_2 + \mu_3. \quad (9)$$

It is interesting, that for  $\delta \ll \eta \ll 1$  the CMM has made the hierarchy much more pronounced, than original situation with  $\mu_1 < \mu_2 < \mu_3$ , and  $m_1, m_2$  can be made very close to zero, while  $m_3$  is not far from  $\mu_3$ .

The diagonalizing matrix  $W$ , defined as  $\hat{\mu} = W^+ \hat{m} W$ , is given in the appendix 2 of [3] together with the resulting CKM matrix  $V_{CKM} = W_u W_d^+$ .

The matrix  $W$  can be conveniently written in terms of elements  $\mu_{ik}$  and  $m_i$ , using the method of [4], in CMM this is simplified in the limit  $\delta \rightarrow 0$ , yielding condition  $W_{31} = 0$ , and an equivalent limiting form for the  $4 \times 4$  matrix  $W$  has three zeros:  $W_{31} = W_{41} = W_{42} = 0$

$$W = \begin{pmatrix} c_\alpha, & -s_\alpha c_\beta e^{i\delta_{12}}, & s_\alpha s_\beta e^{i\delta_{13}} \\ -s_\alpha e^{-i\delta_{12}}, & -c_\alpha c_\beta, & c_\alpha s_\beta e^{i\delta_{23}} \\ 0, & s_\beta e^{-i\delta_{23}}, & c_\beta \end{pmatrix} \quad (10)$$

$W$  is expressed via two sine parameters  $s_\alpha, s_\beta$  and two phases. Here the phases  $\delta_{ik} = \delta_i - \delta_k, k = 1, 2, 3$  satisfy conditions

$$\delta_{12} - \delta_{13} + \delta_{23} = 0. \quad (11)$$

Parameters  $c_\alpha, c_\beta, (s_i = \sqrt{1 - c_i^2}, i = \alpha, \beta)$  are expressed via  $m_i, \mu_i, i = 1, 2, 3$  (similar expressions are found in [4]).

$$c_\alpha = \sqrt{\frac{m_2 - \mu_1}{m_2 - m_1}}, \quad s_\alpha = \sqrt{1 - c_\alpha^2}, \quad c_\beta = \sqrt{\frac{m_3 - \mu_2}{2m_3 - \mu_2 - \mu_3}}, \quad s_\beta = \sqrt{1 - c_\beta^2}. \quad (12)$$

Surprisingly, the simple form (10) as will be seen yields realistic CKM matrix for three generations, which is written below in two ways: a general form, and another with assumption of  $\delta_k^u = \delta_k^d = 0$  for  $k \neq 1$ .

$$V_{ud} = c_\alpha^u c_\alpha^d + c_\beta^u c_\beta^d s_\alpha^u s_\alpha^d e^{i\Delta_{12}} + s_\alpha^u s_\beta^u s_\alpha^d s_\beta^d e^{i\Delta_{13}} \approx c_\alpha^u c_\alpha^d + s_\alpha^u s_\alpha^d e^{i\Delta_{12}} \cos(\beta_u - \beta_d),$$

$$V_{us} = s_\alpha^u c_\beta^u c_\alpha^d c_\beta^d e^{i\delta_{12}^u} - c_\alpha^u s_\alpha^d e^{i\delta_{12}^d} + s_\alpha^u s_\beta^u c_\alpha^d s_\beta^d e^{i\delta_{13}^u - i\delta_{23}^d} \approx -c_\alpha^u s_\alpha^d e^{i\delta_{12}^d} + s_\alpha^u c_\alpha^d e^{i\delta_{12}^u} \cos(\beta_u - \beta_d)$$

$$\begin{aligned}
V_{ub} &= s_\alpha^u s_\beta^u c_\beta^d e^{i\delta_{13}^u} - s_\beta^d s_\alpha^u c_\beta^u e^{i(\delta_{12}^u + \delta_{23}^d)} \approx s_\alpha^u e^{i\delta_{13}^u} \sin(\beta_u - \beta_d) \\
V_{cd} &= -s_\alpha^u c_\alpha^d e^{-i\delta_{12}^u} + c_\alpha^u c_\beta^u c_\beta^d s_\alpha^d e^{-i\delta_{12}^d} + \\
&+ c_\alpha^u s_\beta^u s_\alpha^d s_\beta^d e^{i\delta_{23}^u - i\delta_{13}^d} \approx c_\alpha^u s_\alpha^d e^{-i\delta_{12}^d} \cos(\beta_u - \beta_d) - s_\alpha^u c_\alpha^d e^{-i\delta_{12}^u} \quad (13)
\end{aligned}$$

$$V_{cs} = s_\alpha^u s_\alpha^d e^{-i\Delta_{12}} + c_\alpha^u c_\beta^u c_\alpha^d c_\beta^d + c_\alpha^u c_\alpha^d s_\beta^u s_\beta^d e^{i\Delta_{23}} \approx s_\alpha^u s_\alpha^d e^{-i\Delta_{12}} + c_\alpha^u c_\alpha^d \cos(\beta_u - \beta_d),$$

$$V_{cb} = c_\alpha^u c_\beta^d s_\beta^u e^{i\delta_{23}^u} - c_\alpha^u c_\beta^u s_\beta^d e^{i\delta_{23}^d} \approx c_\alpha^u \sin(\beta_u - \beta_d),$$

$$V_{td} = c_\beta^u s_\alpha^d s_\beta^d e^{-i\delta_{13}^d} - c_\beta^d s_\beta^u s_\alpha^d e^{-i\delta_{23}^u - i\delta_{12}^d} \approx s_\alpha^d e^{-i\delta_{13}^d} \sin(\beta_d - \beta_u),$$

$$V_{ts} = -s_\beta^u c_\alpha^d c_\beta^d e^{-i\delta_{23}^u} + c_\alpha^d c_\beta^u s_\beta^d e^{-i\delta_{23}^d} \approx c_\alpha^d \sin(\beta_d - \beta_u),$$

$$V_{tb} = s_\beta^u s_\beta^d e^{-i\Delta_{23}} + c_\beta^u c_\beta^d \approx \cos(\beta_u - \beta_d).$$

Here  $\Delta_{ij} \equiv \delta_{ij}^u - \delta_{ij}^d$ ,  $\cos \beta_u \equiv c_\beta^u$ ,  $\cos \beta_d \equiv c_\beta^d$ .

3. We compare now the CKM matrix elements with experimental data [5], first assuming three generations and adjusting values of  $\mu_i, m_i$  in  $s_\alpha^u, s_\beta^u, s_\alpha^d, s_\beta^d$ , and  $\Delta_{12}, \Delta_{13}$  to reproduce experimental bounds.

For the CKM matrix (13) one can simplify replacing all  $c_i^s, i = \alpha, \beta, s = u, d$  by unity (this amounts to  $(1 \div 2)\%$  accuracy).

One takes values of  $m_i$  at the scale 2 GeV from [5],  $m_u = 2$  MeV,  $m_d = 5$  MeV,  $m_c = 1.25$  GeV,  $m_s = 95$  MeV,  $m_b = 4.54$  GeV,  $m_t = 173$  GeV.

From (13) one obtains

$$\frac{s_\alpha^u}{s_\alpha^d} = \frac{|V_{ub}|}{|V_{td}|} = 0.485 \pm 0.08, \quad s_\alpha^d = \frac{|V_{td}|}{|V_{ts}|} = 0.21 \pm 0.03, \quad (14)$$

$$s_\alpha^u = \frac{|V_{ub}|}{|V_{cb}|} = 0.095 \pm 0.012,$$

and the last two values are compatible with the first one. Eqs. (14), (12) allow to derive  $\mu_u \cong 13$  MeV,  $\mu_d \cong 9$  MeV.

Next is the case of  $s_\beta^u, s_\beta^d$  which always enter in (13) with the relative phase  $\Delta_{23}$ ,

$$|V_{cb}| \cong |V_{ts}| = |s_\beta^d - e^{-i\Delta_{23}} s_\beta^u| = 0.0412 \pm 0.0011. \quad (15)$$

We assume at this point, that  $\Delta_{23} \equiv 0$  (which yields  $\Delta_{12} = \Delta_{13}$ ),<sup>1</sup> and (15) is satisfied using (12) with  $\mu_s = 0.29$  GeV and  $\mu_c = 12.25$  GeV. This yields  $s_\beta^u = 0.252, s_\beta^d = 0.211$ . Finally,  $\mu_t, \mu_b$  are defined from the CMM condition (9),  $\mu_t \approx 160$  GeV,  $\mu_b \cong 4.2$  GeV.

One can now check all construction computing reparametrization invariant quantities: angles  $\alpha, \beta, \gamma$ , Wolfenstein parameters  $\bar{\rho}, \bar{\eta}$  [6] and Jarlskog parameter  $J$ [7].

From (13)  $\alpha = \arg\left(-\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right) = \Delta_{12} = (99_{-8}^{+13})^\circ$  [5], while

$$\beta = \arg\left(-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right) = \arg(s_\alpha^d - s_\alpha^u e^{-i\Delta_{12}}). \quad (16)$$

Taking central values of  $s_\alpha^d, s_\alpha^u$  from (14) and  $\alpha$  in the experimental bounds, one obtains  $0.67 < \sin 2\beta < 0.8$ , which is compatible with PDG value  $\sin 2\beta = 0.687 \pm 0.0324$ . Another check is calculation of  $\bar{\rho} + i\bar{\eta}$  [6] from (14) and experimental value of  $\alpha = \Delta_{12}$ ,

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} = \frac{s_\alpha^u (s_\alpha^u - s_\alpha^d e^{-i\alpha})}{(s_\alpha^u - s_\alpha^d e^{-i\alpha})^2} = 0.208 + i0.337. \quad (17)$$

This value is well compared with experimental  $(\bar{\rho} + i\bar{\eta})_{exp} = 0.221_{-0.028}^{+0.064} + i0.34_{-0.045}^{+0.017}$ . Finally the Jarlskog parameter for central values from (13), (14) is

$$J = s_\alpha^u s_\alpha^d |s_\beta^d - s_\beta^u e^{i\Delta_{23}}|^2 \sin \Delta_{12} = 3.15 \cdot 10^{-5} \quad (18)$$

vs experimental [5]  $J = (3.08_{-0.18}^{+0.16}) \cdot 10^{-5}$ .

We thus see, that CKM matrix (13) provides a simple and reasonable form in good agreement with experimental data for three generations.

The same is true also for the neutrino mixing matrix  $V_{e\nu}$ , and it is demonstrated in Appendix 2, that  $V_{e\nu}$  acquires the tribimaximal form, when masses

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<sup>1</sup>This assumption implies that the CP violating phase is generated by the interaction of lowest generation with the nontrivial vacuum of the field  $C_\mu$ , and will be discussed at length elsewhere.

$\mu_i$  have reasonable values and  $m_i$  satisfy experimental bounds. The difference from the quark matrices lies in a very small value of  $s_\alpha^e \rightarrow 0$ , ( while  $s_\alpha^d \approx 0.2$ ), which leads to the tribimaximal form of  $V_{e\nu}$ .

As it is, one can persuade oneself, that the last form of the CKM matrix in (13) is defined by three angles and one phase, e.g. by  $s_\alpha^4, s_\alpha^d$  and  $\sin(\beta_u - \beta_d) \equiv s_\beta^u c_\beta^d - s_\beta^d c_\beta^u$ , and the phase  $\Delta_{12} \equiv \alpha$ . The three angle parameters are expressed via  $\mu_u, \mu_d$  and one combination of  $(\mu_c, \mu_s)$  (for  $\sin(\beta_u - \beta_d)$ ). This corresponds to the actual number of parameters in the  $3 \times 3$  unitary matrix, but in our case all parameters have a clear physical meaning.

4. We turn now to the case of four generations.

First of all one realizes, that the Pagels-Stokar relation [8],[1] for our multiple stationary solutions  $\mu_i(p)$  of Eq. (5) analyzed in [3], has a generalized form

$$v^2 = \frac{N_c}{4\pi^2} \sum_{i=1}^4 \mu_i^2 \ln \frac{M}{\mu_i}, \quad (19)$$

where  $v = 246$  GeV and  $\mu_i = \bar{\mu}_i(p)$  is the effective value of  $\mu_i(p)$  in the corresponding loop integral. Since one assumes that  $M \gg \max \mu_i = \mu_4$  the value of  $\mu_4$  is limited from above and for  $M \geq 4\mu_4$  one has  $\mu_4 \lesssim 0.8$  TeV. Now one can connect  $\mu_i$  and  $m_i$  in the same way, as it was done above for three generations (see appendix 3 of [3] for details). One has

$$m_4 = \frac{a_1}{2} + \sqrt{\frac{a_1^2}{4} - a_2}, \quad m_3 = \frac{a_1}{2} - \sqrt{\frac{a_1^2}{4} - a_2}, \quad a_1 = \sum_{i=1}^4 \mu_i, \quad (20)$$

$$a_2 \cong \mu_4(\mu_2 + \mu_3)\bar{\eta} + O(\eta, \delta).$$

here  $\bar{\eta} = -\eta_{34} > 0$ , and if  $\bar{\eta} \gg \frac{1}{4}\eta$ , then relations (9) for  $m_1, m_2$  are approximately the same, so that we keep the values  $\mu_s, \mu_c, \mu_u, \mu_d$  of the previous section.

We take two estimates for  $\mu_3^u, \mu_4^u$ : a)  $\mu_4^u = 0.8$  TeV,  $\mu_3^u = 0.4$  TeV; b)  $\mu_4^u, \mu_3^u = (0.5, 0.3)$  TeV and obtain  $\bar{\eta}_4, s_4^u$  in two cases.

a)  $\bar{\eta}_4 = 0.55, s_4^u = 0.51$ ; b)  $\bar{\eta}_4 = 0.7, s_4^u = 0.54$ . For  $\mu_3^d, \mu_4^d$  the situation is less constrained, since one can choose  $\mu_3^d \ll \mu_4^d$  and one has  $s_4^d \ll 1, \bar{\eta}_d \ll 1$  and an equivalent limiting form for the  $4 \times 4$  matrix  $W$  has three zeros:  $W_{31} = W_{41} = W_{42} = 0$

$$W = \begin{pmatrix} c_\alpha, & -s_\alpha c_\beta e^{i\delta_{12}}, & c_4 s_\alpha s_\beta e^{i\delta_{13}}, & s_4 s_\beta s_\alpha e^{i\delta_{14}} \\ -s_\alpha e^{-i\delta_{12}}, & -c_\alpha c_\beta, & c_4 s_\beta c_\alpha e^{i\delta_{23}}, & s_4 s_\beta c_\alpha e^{i\delta_{24}} \\ 0, & s_\beta e^{-i\delta_{23}}, & c_4 c_\beta, & s_4 c_\beta e^{i\delta_{34}} \\ 0, & 0, & -s_4 e^{-i\delta_{34}}, & c_4 \end{pmatrix}. \quad (21)$$

Here the phases  $\delta_{ik}$  satisfy conditions

$$\delta_{24} = \delta_{23} + \delta_{34}, \quad \delta_{14} = \delta_{13} + \delta_{34}, \quad \delta_{14} - \delta_{24} = \delta_{12}. \quad (22)$$

The  $3 \times 3$  matrix  $W$  is easily obtained putting  $s_4 = 0, c_4 = 1$ .

Parameters  $c_\alpha, c_\beta, c_4 (s_i = \sqrt{1 - c_i^2}, i = \alpha, \beta, 4)$  are expressed via  $m_i, \mu_i, i = 1, 2, 3, 4$

$$c_\alpha^2 = \frac{m_2 - \mu_1}{m_2 - m_1}, \quad c_\beta^2 = \frac{m_3 - \mu_2}{m_3 + \mu_1 - m_1 - m_2}, \quad c_4^2 \cong \frac{\mu_4 - m_3 + \mu_2}{m_4 - m_3 + \mu_2}. \quad (23)$$

The resulting CKM matrix is given in Appendix 1.

One can check, whether CKM matrix is compatible with experiment in the case of four generations. The sensitive points are unitarity relations and the matrix element  $V_{ud}$ .

We list as an example two possible sets of masses  $\mu_i$  which define together with  $m_i$  the parameters  $s_k^i, c_k^i, i = u, d; k = \alpha, \beta, 4$

$\mu_d = 10$  MeV,  $\mu_s = 0.29$  GeV,  $\mu_u = 13$  MeV,  $\mu_c = 12.25$  GeV;

set a)  $\mu_t, \mu_{t'} = (400, 800)$  GeV;  $\mu_b, \mu_{b'} = (20, 400)$  GeV;  $m_{b'}, m_{t'} = (420, 1200)$  GeV;

set b)  $\mu_t, \mu_{t'} = (300, 500)$  GeV;  $\mu_b, \mu_{b'} = (200, 380)$  GeV;  $m_{b'}, m_{t'} = (580, 630)$  GeV.

The resulting parameters are:

$s_\beta^u; c_\beta^u = 0.252; 0.968; s_\alpha^u; c_\alpha^u = 0.095; 0.995$

$s_\beta^d; c_\beta^d = 0.211; 0.98; s_\alpha^d; c_\alpha^d = 0.21; 0.98;$

a)  $s_4^u; c_4^u = 0.51; 0.86; s_4^d; c_4^d = 0.22, 0.97;$

b)  $s_4^u; c_4^u = 0.54; 0.85; s_4^d; c_4^d = 0.57, 0.82.$

Let us first check, whether the  $3 \times 3$  part of the CKM matrix is not deteriorated by the inclusion of the 4-th generation.

Taking  $V_{ud}$  and parameter values from Appendix 1, one can write

$$V_{ud} = 0.975 + 0.019 \cdot e^{i\Delta_{12}} + 0.00084 e^{i\Delta_{13}} + 1.2 \cdot 10^{-4} e^{i\Delta_{14}}. \quad (24)$$



Assuming as before, that the nonzero phase is due to the first generation only, i.e.  $\Delta_{12} = \Delta_{13} = \Delta_{14} = \alpha$  and  $\alpha$  in experiment is close to  $\frac{\pi}{2}$  [5], one obtains a good agreement with experimental value for  $V_{ud}$ ,  $|V_{ud}| = 0.97418 \pm 0.00027$  [5] with strong limits on  $\alpha$ ,  $|\alpha - 90^\circ| < 1.5^\circ$ .

Another CKM element known with good accuracy is  $V_{us}$  and with the same parameters from Appendix 1 one obtains for  $\Delta_{12} = \alpha = 90^\circ$ , and the same assumptions about  $\Delta_{ik} = 0$  with  $i, k \neq 1, |V_{us}| = 0.228$ , which is close to the experimental value [5]:  $|V_{us}| = 0.2255 \pm 0.0019$ , and  $|V_{cd}| = 0.215$  vs experimental  $|V_{cd}| = 0.230 \pm 0.011$  [5]. In a similar way one can check other CKM coefficients which are known experimentally with larger errors and therefore this check is not sensitive to the contribution of the fourth generation.

5. At this point one can compare our results with that in the literature, for reviews and references see [9, 10].

First of all, we note, that our  $4 \times 4$  CKM matrix contains two additional phases and two angles ( $s_4^u, s_4^d$ ) as compared to the  $3 \times 3$  matrix. Moreover, in the one-phase limit ( $\Delta_{12} = \Delta_{13} = \Delta_{14}$ ,  $\Delta_{ik} = 0, i, k \neq 1$ ) no additional phase appears, and  $s_4^u, s_4^d$  and hence all  $V_{ij}$  are defined by the masses  $\mu_4^{u,d}, m_4^{u,d}$ , in addition to four parameters of  $3 \times 3$  matrix, and the resulting scheme is rather rigid with six overall parameters, which is less than 9 possible parameters of  $4 \times 4$  unitary matrix (with  $2n - 1$  phases removed).

First of all, it is important to compare our results with the bounds [11]-[14] on the mixing coefficients and masses  $m_{b'}, m_{t'}$ , following from FCNC box diagrams and the decay  $b \rightarrow s\gamma$ , and also from the so-called precise EW tests (EWPT). The FCNC bound on mixing coefficients  $V_{ib'}, V_{t'i}$ , were studied in [14], and can be written in our terms (assuming as before the only nonzero CPV phase  $\Delta_{1i} = \alpha$ ) as the bound on the combination  $K = -c_4^u s_4^d + s_4^u c_4^d < 0.6$  (for a “conservative bound” [14]). From our mixing coefficients above one has  $K \cong 0.3$  for the set a) and  $K = 0.042$  for the symmetric set b). Thus both sets satisfy the FCNC bounds of [14]. However, as shown in [13], the EWPT bounds are much more restrictive, and can be again reduced to the bound on the same quantity  $K (\cong s_{34}$  in notations of [13]), e.g.  $K \lesssim 0.1$  (95% C.L.). One can see, that this limit is easily satisfied by our set b), where masses  $m_{b'} \approx m_{t'} \approx 0.6$  TeV and  $m_{t'} - m_{b'} \approx 50$  GeV, in the same range, as the values considered in [12, 14]. Thus one can see, that the mixing  $V_{ib'} \simeq K \approx -V_{t'b} \approx 0.1$  can be easily accommodated in the coherent mixing scheme, and it satisfies both FCNC and EWPT bounds. On another hand, as it is stressed in [15], this mixing opens up an interesting possibility of the

search for the 3-body decay of Higgs in processes like  $H \rightarrow \bar{t}bW^+$  or  $\bar{b}tW^-$ , where the wide composite Higgs is in the TeV region.

Summarizing, a new mechanism producing several generations of quarks, and mixing between them is suggested. The resulting CKM matrix in agreement with all experimental data is found for three generations. In the case of four generations stringent FCNC and EWPT bounds on masses and mixings drastically limit available space of mass parameters. It is also interesting, that our scheme provides a simple acceptable parametrization of the lepton mixing matrix, as shown in Appendix 2.

As it was stressed recently in [16], in the general situation with  $m_{\nu'} \neq \frac{1}{2}m_{Z'}$ , the case of four generations is still disfavored in the global fit analysis, including oblique parameters  $S, T, U$  and FCNC constraints. For possible implications of the latter see [17].

Our scheme predicts the EWSB connected primarily with  $b, t$  quarks in case of 3 and with the  $b', t'$  quarks in case of four generations (see e.g. Eq. (19)), and this is in common with approaches derived in [18].

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## Appendix 1

## The CKM $4 \times 4$ matrix elements

The CKM matrix  $V = W_u W_d^+$  is obtained from (10) with parameters  $s_\alpha^u, s_\beta^u, s_4^u, s_\alpha^d, s_\beta^d, s_4^d$  (and the corresponding cosine terms), and phases  $\delta_{ij}^u, \delta_{ij}^d$ , satisfying conditions (11) and (22).

$$V_{ud} = c_\alpha^u c_\alpha^d + s_\alpha^u s_\alpha^d c_\beta^u c_\beta^d e^{i\Delta_{12}} + c_4^u s_\alpha^u s_\beta^u c_4^d s_\beta^d s_\alpha^d e^{i\Delta_{13}} + s_4^u s_4^d s_\beta^u s_\alpha^u s_\beta^d s_\alpha^d e^{i\Delta_{14}};$$

$$V_{us} = -c_\alpha^u s_\alpha^d e^{i\delta_{12}^d} + s_\alpha^u c_\beta^u c_\alpha^d c_\beta^d e^{i\delta_{12}^u} + c_4^u c_4^d s_\alpha^u s_\beta^u s_\beta^d c_\alpha^d e^{i\delta_{13}^u - i\delta_{23}^d} + s_4^u s_4^d s_\beta^u s_\alpha^u s_\beta^d c_\alpha^d e^{i\delta_{14}^u - i\delta_{24}^d}$$

$$V_{ub} = -s_\alpha^u c_\beta^u s_\alpha^d e^{i\delta_{12}^u + \delta_{23}^d} + c_4^u c_4^d s_\alpha^u s_\beta^u c_\beta^d e^{i\delta_{12}^u} + s_4^u s_4^d s_\beta^u s_\alpha^u c_\beta^d e^{i\delta_{14}^u - i\delta_{34}^d}$$

$$V_{ub'} = -c_4^u s_4^d s_\alpha^u s_\beta^u e^{i\delta_{13}^u + i\delta_{34}^d} + s_4^u c_4^d s_\beta^u s_\alpha^u e^{i\delta_{14}^u}$$

$$V_{cd} = -s_\alpha^u c_\alpha^d e^{-i\delta_{12}^u} + c_\alpha^u c_\beta^u s_\alpha^d c_\beta^d e^{-i\delta_{12}^d} + c_4^u c_4^d s_\beta^u c_\alpha^u s_\alpha^d s_\beta^d e^{i\delta_{23}^u - i\delta_{13}^d} + s_4^u s_4^d s_\beta^u c_\alpha^u s_\beta^d s_\alpha^d e^{i\delta_{24}^u - i\delta_{14}^d} \quad (\text{A1.1})$$

$$V_{cs} = s_\alpha^u s_\alpha^d e^{-i\Delta_{12}} + c_\alpha^u c_\beta^u c_\alpha^d c_\beta^d + c_4^u c_4^d s_\beta^u c_\alpha^u s_\beta^d c_\alpha^d e^{i\Delta_{23}} + s_4^u s_4^d s_\beta^u c_\alpha^u s_\beta^d c_\alpha^d e^{i\Delta_{24}}$$

$$V_{cb} = -c_\alpha^u c_\beta^u s_\beta^d e^{i\delta_{23}^d} + c_4^u c_4^d s_\beta^u c_\alpha^u c_\beta^d e^{i\delta_{23}^u} + s_4^u s_4^d s_\beta^u c_\alpha^u c_\beta^d e^{i\delta_{24}^u - i\delta_{34}^d},$$

$$V_{cb'} = -c_4^u s_4^d s_\beta^u c_\alpha^u e^{i\delta_{23}^u + i\delta_{34}^d} + s_4^u c_4^d s_\beta^u c_\alpha^u e^{i\delta_{24}^u}$$

$$V_{td} = -s_\beta^u s_\alpha^d c_\beta^d e^{-i\delta_{23}^u + i\delta_{12}^d} + c_4^u c_4^d c_\beta^u s_\alpha^d s_\beta^d e^{-i\delta_{13}^d} + s_4^u s_4^d c_\beta^u s_\beta^d s_\alpha^d e^{i\delta_{34}^u - i\delta_4^d},$$

$$V_{ts} = -s_\beta^u c_\alpha^d c_\beta^d e^{-i\delta_{23}^u} + c_4^u c_4^d c_\beta^u s_\beta^d c_\alpha^d e^{-i\delta_{23}^d} + s_4^u s_4^d c_\beta^u s_\beta^d c_\alpha^d e^{i\delta_{34}^u - i\delta_{24}^d},$$

$$V_{tb} = s_\beta^u s_\beta^d e^{-i\Delta_{23}} + c_4^u c_4^d c_\beta^u c_\beta^d + s_4^u s_4^d c_\beta^u c_\beta^d e^{i\Delta_{34}},$$

$$V_{tb'} = -c_4^u s_4^d c_\beta^u e^{i\delta_{34}^d} + s_4^u c_4^d c_\beta^u e^{i\delta_{34}^u}$$

$$V_{\nu'd} = -s_4^u c_4^d s_\alpha^d s_\beta^d e^{-i\delta_{34}^u - i\delta_{13}^d} + c_4^u s_4^d s_\beta^d s_\alpha^d e^{-i\delta_{14}^d}$$

$$V_{\nu's} = -s_4^u c_4^d s_\beta^d c_\alpha^d e^{-i\delta_{34}^u - i\delta_{23}^d} + c_4^u s_4^d s_\beta^d c_\alpha^d e^{-i\delta_{24}^d}$$

$$V_{\nu'b} = -s_4^u c_4^d c_\beta^d e^{-i\delta_{34}^u} + c_4^u s_4^d c_\beta^d e^{-i\delta_{34}^d}$$

$$V_{\nu'b'} = s_4^u s_4^d e^{-i\Delta_{34}} + c_4^u c_4^d$$

Parameter values  $s_\alpha^u, s_\alpha^d, s_\beta^u, s_\beta^d$  were discussed in the text, Eq. (23) and below.

## Appendix 2

### Coherent mixing in leptons

In this appendix we apply the CMM matrices to families of  $e$ - and  $\nu$ -sectors. As in the main text and in [3] we define the  $\hat{W}_e, \hat{W}_\nu$  matrices in the following way

$$\hat{\mu}_{e,\nu} = \hat{W}_{e,\nu}^+ m_{e,\nu} (diag) \hat{W}_{e,\nu} \quad (\text{A2.1})$$

so that the mixing matrix is  $\hat{V}_{e,\nu} = \hat{W}_e \hat{W}_\nu^+$ .

In the  $e$ -sector with masses  $m_e = 0.51$  MeV,  $m_\mu = 105.66$  MeV and  $m_\tau = 1777$  MeV.

Choosing  $\mu_e \cong 1$  MeV, and not fixing yet  $\mu_\mu$  and  $\mu_\tau$ , one obtains the matrix  $\hat{W}_e$  in the simplified form (10), where we put  $c_\alpha \rightarrow c_\alpha^e$ ,  $s_\alpha \rightarrow s_\alpha^e$ ,  $c_\beta \rightarrow c_\beta^e$ ,  $s_\beta \rightarrow -s_\beta^e$ . We shall neglect for simplicity possible Majorana phases, and obtain using (12)

$$s_\alpha^e \cong 0.07 \rightarrow 0, \quad c_\alpha^e \cong 1; \quad s_\beta^e, \quad c_\beta^e. \quad (\text{A2.2})$$

In the same way we are obtaining the matrix  $\hat{W}_\nu$  as in (12) replacing  $s_\beta$  in (10) by  $(+s_\beta^\nu)$ , and we have

$$V_{e\nu}(s_\alpha^e \rightarrow 0) = \begin{pmatrix} c_\alpha^\nu & s_\alpha^\nu e^{i\delta_{12}^\nu} & 0 \\ -s_\alpha^\nu \cos(\beta_e + \beta_\nu) e^{-i\delta_{12}^\nu} & c_\alpha^\nu \cos(\beta_e + \beta_\nu) & -\sin(\beta_e + \beta_\nu) \\ -s_\alpha^\nu \sin(\beta_e + \beta_\nu) e^{-i\delta_{13}^\nu} & c_\alpha^\nu \sin(\beta_e + \beta_\nu) & \cos(\beta_e + \beta_\nu) \end{pmatrix}, \quad (\text{A2.3})$$

where  $\cos \beta_e \equiv c_\beta^e$ ,  $\cos \beta_\nu \equiv c_\beta^\nu$ , and using experimental data [5],  $\sin \theta_{13} < 5.10^{-2}$ ,  $\sin^2(2\theta_{23}) > 0.90$ ,  $tg^2 \theta_{12} = 0.47_{-0.05}^{+0.06}$ , one has

$$s_\alpha^\nu \cong \frac{1}{\sqrt{3}}, \quad c_\alpha^\nu \cong \sqrt{\frac{2}{3}}, \quad \beta_e + \beta_\nu \cong \frac{\pi}{4} \quad (\text{A2.4})$$

In this way one reproduces the well-known tribimaximal matrix  $V_{e\nu}(s_\alpha^e \rightarrow 0) = U_{TB}$ , where

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (\text{A2.5})$$

At this point one should find  $\mu_\mu, \mu_\tau$  and the set  $\mu_i^\nu$ ,  $i = 1, 2, 3$ , which correspond to the conditions (A2.4). Having in mind, that  $s_\beta^e = \sqrt{\frac{m_\tau - \mu_\tau}{2m_\tau - \mu_\tau - \mu_\mu}} \approx \sqrt{\frac{\mu_\mu}{m_\tau}} \geq \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.25$  and  $s_\beta^\nu \geq \sqrt{\frac{m_2}{m_3}} \approx 0.42$ , (for experimental values [5]  $m_2 \approx 0.875 \cdot 10^{-2}$  eV,  $m_3 \approx 4.9 \cdot 10^{-2}$  eV, and assuming  $m_{2,3} \gg m_1$ ) one has a narrow interval,  $\beta_e \gtrsim 15^\circ$ ,  $\beta_\nu \gtrsim 25^\circ$ ,  $\beta_e + \beta_\nu = 45^\circ$ , and  $\mu_3^\nu$  varies in the interval  $0.75m_3 \leq \mu_3^\nu \leq 0.82m_3$ ;  $m_\mu \leq \mu_\mu \leq 2m_\mu$ . For the first generation of  $\nu$  both  $\mu_1^\nu > m_1$  should only be much smaller, than  $m_2, m_3$ .

For more discussion with the use of  $W$  matrices of the type of Eq. (10) see [4].